PARTICLE DEPOSITION ON CHANNEL WALLS IN A TURBULENT FLOW

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The process of admixture deposition from a turbulent flow for different values of the particle reflection coefficient from the wall is analyzed. A generalizing formula is proposed for the computation of the deposition rate on a completely absorbing surface in a broad range of variation of the particle inertia.

A sufficiently significant quantity of publications, [1-5], say, is devoted to the computation of particle deposition on the surface of channels in turbulent flows. However, the complete theory that takes account sequentially of the different deposition mechanisms as a function of the particle inertia has not yet been constructed. In practice, there is not any analysis even of the influence of conditions of particle interaction with the wall (boundary conditions on the surface) on the deposition intensity. Certain models are examined in this paper for the computation of the deposition in a broad range of variation of the particle size for different conditions of their interaction with the wall, taken into account by the reflection coefficient.

The following system of mass and momentum balance equations for a solid disperse phase is obtained in [5] from the kinetic equation constructed for the probability density of the particle coordinate and velocity distributions in a turbulent flow

$$\frac{\partial C}{\partial t} + \frac{\partial}{\partial x_k} CV_k = 0, \tag{1}$$

$$\frac{\partial V_i}{\partial t} + V_k \frac{\partial V_i}{\partial x_k} = -\frac{\partial \langle v_i' v_k' \rangle}{\partial x_k} + \frac{U_i - V_i}{\tau} - \frac{D_{ik}}{\tau} \frac{\partial \ln C}{\partial x_k}. \tag{2}$$

Here $\langle v_i' v_k' \rangle$ is the stress tensor in the solid phase that occurs because of particle involvement in the turbulent motion of the carrying gas flow and Brownian motion, and $D_{ik} = \tau [\langle v_i' v_k' \rangle + (T/\tau - f) \langle u_i' u_k' \rangle]$ is the diffusion tensor. Without taking account of inhomogeneities of the average and fluctuating velocity fields described by the convective and diffusion terms in the balance equations of the second moments of the solid phase velocity fluctuations [5], the quantity $\langle v_i' v_k' \rangle$ is determined by the expression

$$\langle v_i' v_k' \rangle = f \langle u_i' u_k' \rangle + \frac{D}{\tau} \delta_{ik}, \tag{3}$$

where the coefficient f characterizes the degree of particle involvement in the fluctuating motion and has the form $f = 1 - \exp(-T/\tau)$ [6].

As the particle inertia increases, that is characterized by the parameter τ/T, the accuracy of the equilibrium (locally homogeneous) relationship (3) is lowered. This circumstance appears to be especially noticeable near the channel wall, where the flow becomes substantially inhomogeneous on the one hand since there are large average velocity and turbulent energy gradients, and the particles turn out to be relatively more coarse because of the diminution in the time scale of the turbulence T on the other.

Taking account of (3) the following equation for the particle concentration distribution over the channel section is obtained from (1) and (2) for hydrodynamically developed stationary flow

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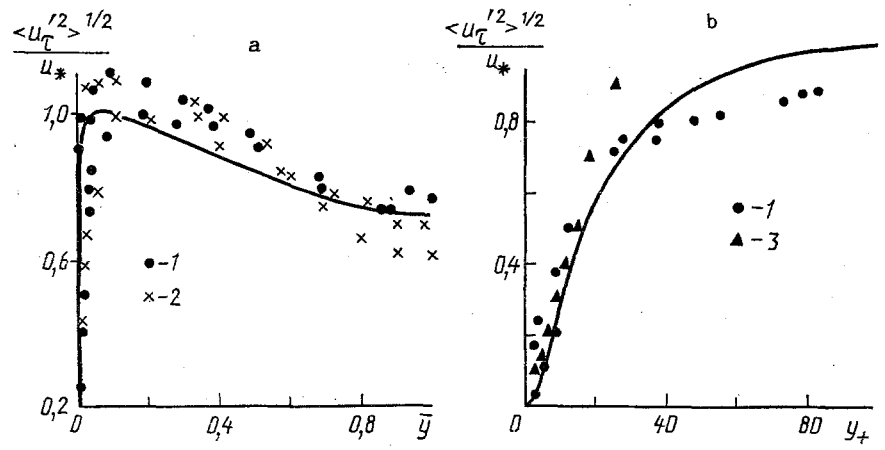

Fig. 1. Distribution of r.m.s. fluctuations of the radial gas velocity component over the channel section $\langle u_r'^2\rangle^{1/2}/u_*$ in the near-wall domain (a) and in the flow core (b): 1) [10]; 2) [11]; 3) [12].

$$(D_T + D)\frac{\partial C}{\partial r} + \frac{\partial (qD_T)}{\partial r} C = -\frac{r}{R} J_w, \tag{4}$$

where $D_T = T\langle u_r'^2\rangle$ is the diffusion coefficient and $q = \tau f/T$ is the migration coefficient.

For inertial particles the diffusion coefficient in (4) agrees with the diffusion coefficient for an inertialess admixture, which is in agreement with the known Chen theorem [7]. However, in contrast to the ordinary diffusion equation for an inertialess admixture there is an additional migration term in (4) due to the inhomogeneity of the field of turbulent carrier flow fluctuations; where as the particle inertia grows the role of the migration transfer mechanism rises and the migration coefficient q grows from zero to one as the ratio τ/T increases.

The turbulent diffusion coefficient in (4) is expressed in terms of the carrier flow turbulent viscosity coefficient $D_T = \nu_T/Sc_T$, where $Sc_T = 0.9$. The coefficient of turbulent viscosity is determined from the formula

$$\frac{\nu_T}{\nu} = \frac{1}{6}\left\{\sqrt{1 + 4[1 - \exp(-y_+/A)]^2\varkappa^2 y_+^2} - 1\right\}(2 - \bar{y})\left[\frac{1}{2} + (1 - \bar{y})^2\right], \tag{5}$$

which goes over into the Reichardt formula far from the wall and into the van Driest relationship [8] ($\kappa = 0.4$; $A = 26$) as $\bar{y} \to 0$.

The integral time scale of turbulence needed to calculate the migration coefficient q is given by the interpolation formula

$$T_+ = Tu_*^2/\nu = \sqrt{T_{+0}^2 + l_+^2}, \tag{6}$$

that goes over into the relationship $T_+ = T_{+0} = \text{const}$ as $y_+ \to 0$ (the value of the time scale of turbulence is ordinarily assumed constant in the domain of the viscous sublayer, for instance [9]) and into the relationship $T_+ = l_+ = lu_*/\nu$ far from the wall, where l is the mixing path length. The constant T_{+0} is assumed to equal 10 while the quantity l_+ is determined from the Prandtl-Nikuradze formula

$$l_+ = \varkappa y_+(1 - 1{,}1\bar{y} + 0{,}6\bar{y}^2 - 0{,}15\bar{y}^3).$$

In order to confirm the approximation selected for T, the expression for the r.m.s. fluctuations of the radial gas velocity component $\langle u_r'^2\rangle = \nu_T/TSc_T$ obtained from the relation $D_T = \nu_T/Sc_T = T\langle u_r'^2\rangle$ is compared with experimental data presented in [10-12]. It is seen that the results of the computations obtained by using (6) agree sufficiently well with the experimental data in both the near-wall domain and in the flow core.

The boundary condition on the surface is obtained for (4) from the solution of the kinetic equation for the probability density in the near-wall domain and has the form [5]

$$C_w = \frac{1+\chi}{1-\chi}\left(\frac{\pi}{2\langle v_r'^2\rangle_w}\right)^{1/2} J_w, \tag{7}$$

where the reflection coefficient χ is defined as the ratio between the reflected and incident particle fluxes. Therefore, from the viewpoint of particle reflection or absorption the physical properties of the surface are characterized by the coefficient χ equal to the probability of recoil and return of particles colliding with the wall to the flow.

It follows from (4)-(7) that the coefficient of deposition $j_+ = J_w/u_*C_0$ can be represented in the form of a functional dependence on the governing parameters as follows

$$j_+ = F(\tau_+, B, R_+, \chi),$$

where $\tau_+ = \tau u_*^2/\nu$ characterizes the particle inertia, $B = (9\rho_1/2\rho_2)^{1/2}6\pi\rho_1\nu^3/k\Theta u_*$ is the influence of Brownian diffusion on deposition (the parameter B appers from the relationship for the Schmidt number $Sc = \nu/D = \tau_+^{1/2}B$); $R_+ = Ru_*/\nu$ is the flow Reynolds number. The parameter τ_+ is the principal one, its value determines the role of the different mechanisms in the deposition process; the influence of the other parameters depends on the range of variation of τ_+ and in certain domains of values of τ_+ can be lacking in practice.

Let us first examine the case of very fine particles ($\tau_+ \ll 1$), when the influence of turbulent migration on the deposition can be neglected (i.e., we can set $q = 0$ in (4)) and a pure diffusion mode of deposition is realized. Because of the large values of the Schmidt number ($Sc \gg 1$), a change in the concentration occurs in a thin diffusion layer whose thickness is much less than the dimension of the viscous sublayer. In this case, taking account of (5) and (7) there follows from (4)

$$j_+ = \frac{\left(\frac{\varkappa^2}{Sc_T A^2}\right)^{1/4} \frac{2^{3/2}}{\pi\tau_+^{3/2} B^{3/4}}}{1 + \frac{1+\chi}{1-\chi}\left(\frac{\varkappa^2}{Sc_T A^2}\right)^{1/4} \frac{2\tau_+^{3/4}}{\pi^{1/2} B^{1/4}}}. \tag{8}$$

Without taking account of the second term in the denominator, (8) agrees exactly with the formula presented in [13] for the mass delivery of an inertialess admixture for large Schmidt numbers

$$j_+ = \left(\frac{\varkappa^2}{Sc_T A^2}\right)^{1/4} \frac{2^{3/2}}{\pi\tau_+^{3/8} B^{3/4}} = \frac{0{,}115}{Sc^{3/4}}. \tag{9}$$

It is seen from (8) that the quantity j_+ is independent of the parameter R_+. Moreover, since $B \gg 1$ under real conditions, while $\tau_+ \ll 1$ in the case under consideration, then the influence of the reflection coefficient on the coefficient of deposition j_+ appears only for values of χ quite close to one. Therefore, for very fine particles on a non-reflecting surface ($\chi \neq 1$) the dependence $j_+ = F(\tau_+, B)$ holds.

The results of a numerical solution of the problem (4)-(7) are represented in Fig. 2 in a sufficiently broad range of variation of the particle inertia for a circular channel; experimental data of different authors assembled in [14] are presented there. Since values of the turbulent gas stresses (and particularly the quantity $\langle u_r'^2\rangle$ governing the diffusion coefficient D_T) become quite small in the viscous sublayer domain while the fluctuating solid phase energy can correspond to the value at a certain distance from the wall because of the particle inertial path, i.e., can exceed the value given by (3) substantially, the utilization of the relationship (3) obtained for a homogeneous turbulent flow and being local in nature can result in incorrect results, in principle, near the wall. In order to take effective account of the inertial transfer mechanism of the fluctuating energy in the solid phase, we will give the boundary condition (7) at a certain distance from the wall y_w proportional to the length of the inertial path of the particles [15] $y_w = \alpha u_*\tau$, where α is a constant. Computation directly at the wall results in sharp peaks in the concentration in the viscous sublayer which have no physical meaning.

Results of a computation for a completely absorbing surface ($\chi = 0$) are shown for $\alpha = 0.5$ by solid lines in Fig. 2. It is seen that satisfactory agreement between the computed and experimental data hold in the range of variation of the particle inertia under consideration. The initial drop in deposition intensity as τ_+ grows is related, according to (9), to the diminution in the Brownian diffusion coefficient. The rise in the coefficient of deposition as τ_+ grows for relatively inertial particles ($\tau \gtrsim 0.1$) is explained by the

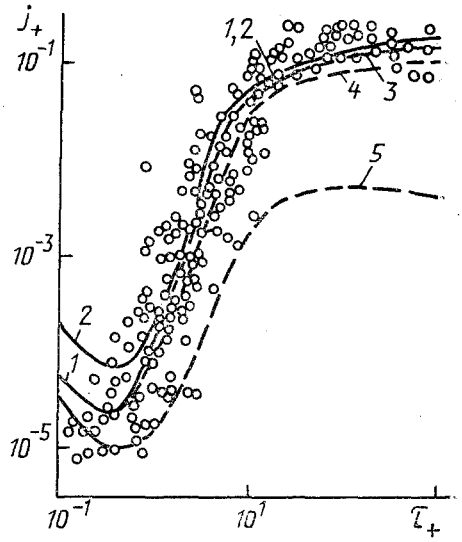

Fig. 2. Results of a numerical computation of the coefficient of deposition ($R_+ = 5\cdot10^4$): 1, 3, 4, 5) $B = 10^6$; 2) $2\cdot10^5$; 1, 2) $\chi = 0$; 3) 0.4; 4) 0.8; 5) 0.99.

increase in the role of the migration transfer mechanism. The contribution of Brownian diffusion in this domain turns out to be negligibly small and, therefore, the quantity B is excluded from the number of governing parameters. Therefore, the mechanism of inertial particle deposition is determined by turbulent diffusion and turbulent migration processes due to inhomogeneity in the distributions of the concentration and velocity fluctuation fields. For not very coarse particles, when the condition $\tau_+ \ll R_+$ is satisfied, i.e., when the particle inertial path length is much less than the channel radius, the quantity R_+ is not a governing parameter and the coefficient of deposition is determined by the dependence $j_+ = F(\tau_+, \chi)$. In this domain of variation of τ_+ the results of a computation for a completely absorbing wall can be approximated by the formula

$$j_+ = 2{,}5\cdot10^{-4}\tau_+^{2{,}5}. \qquad (10)$$

When the quantity τ_+ takes on values commensurate with R_+, the influence of the parameter R_+ becomes substantial and the model of the computation based on the representation $y_+ = \sigma\tau_+$ ceases to be suitable.

The behavior of the coefficient of deposition as a function of the value of the particle reflection coefficient is shown by dashed lines in Fig. 2. Attention is turned to the weak influence of χ on j_+ in the domain of small values of τ_+, which is in agreement with the solution (8) for very fine particles. As the particle inertia grows the influence of the parameter χ on the deposition increases and the maximum in the dependence $j_+(\tau_+)$ shifts towards lower values of τ_+. Such a nature of the dependence of j_+ on χ has an obvious physical meaning, only inertial particles being reflected from the surface can leave the near-wall domain while fine particles must remain equally in the near-wall layer and, therefore, inevitably precipitate on the wall.

Let us consider the deposition of coarse (very inertial) particles for $\tau_+ \gg 1$. It follows from an analysis of the characteristic scales in this case that the viscosity ν ceases to be a governing parameter and, therefore, not the quantity τ_+ but rather $\tau_0 = \tau_+/R_+ = \tau u_*/R$ should be the parameter characterizing the particle inertia. In this case the stresses in the solid phase are practically invariant along the channel length and can be found from the condition of integral balance between the second velocity fluctuation moments of the disperse and carrying phases

$$\tau\int_0^1 \langle v_i' v_k' \rangle \bar{r}\, d\bar{r} = \int_0^1 T \langle u_i' u_k' \rangle \bar{r}\, d\bar{r},$$

that replaces the local relationship (3) with $f \approx T/\tau$ taken into account for $\tau/T \gg 1$. For $\tau/T \gg 1$ the particle transfer equation (4) reduces to the form

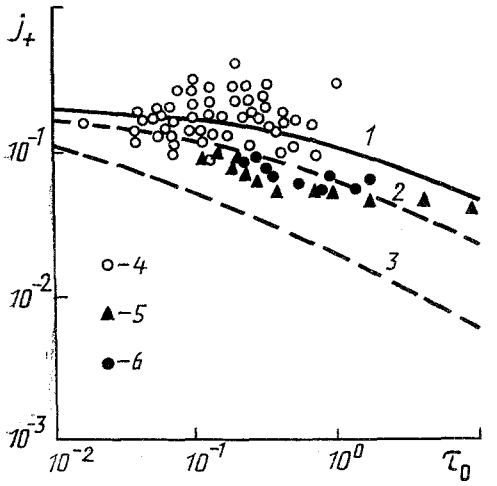

Fig. 3. Coefficient of deposition of coarse particles: 1-3) computation using (12); 1) $\chi = 0$; 2) 0.4; 3) 0.8; 4) [14]; 5) [16]; 6) [17].

$$D_{T0}\frac{dC}{dr} = -\frac{r}{R}J_w. \tag{11}$$

Here

$$D_{T0} = \tau\langle v_r'^2\rangle_0 = 2\int_0^1 T\langle u_r'^2\rangle\bar{r}d\bar{r} = 2\int_0^1 D_T\bar{r}d\bar{r},$$

where according to (5)

$$D_T = \frac{\varkappa u_* R}{3\,\mathrm{Sc}_T}(1-\bar{r}^2)\left(\frac{1}{2}+\bar{r}^2\right).$$

Integrating (11) with the boundary condition (7) taken into account we obtain

$$j_+ = \frac{5\varkappa/9\,\mathrm{Sc}_T}{1+\dfrac{1+\chi}{1-\chi}\left(\dfrac{10\pi\varkappa}{9\,\mathrm{Sc}_T}\tau_0\right)^{1/2}}. \tag{12}$$

A comparison between (12) and experimental data [14, 16, 17] on a completely absorbing surface, i.e., when (12) takes the form

$$j_+ = \frac{5\varkappa/9\,\mathrm{Sc}_T}{1+(10\pi\varkappa/9\,\mathrm{Sc}_T)^{1/2}\tau_0^{1/2}} = \frac{0{,}25}{1+1{,}25\tau_0^{1/2}}, \tag{13}$$

is shown by continuous line in Fig. 3.

It is seen that (13) agrees satisfactorily with the experimental data. The influence of the reflection coefficient on the deposition, which is substantial in contrast to the case of fine particles, is shown by dashed curves. Thus the deposition of coarse particles is described by the dependence $j_+ = F(\tau_+/R_+, \chi)$.

On the whole, the analysis performed indicates that there are three characteristic deposition domains: fine, medium, and coarse particles. The coefficient of deposition on a completely absorbing surface is described in these domains, respectively, by (9), (10), and(13). The following approximate formula

$$j_+ = \frac{\dfrac{0{,}115}{B^{3/4}\tau_+^{3/8}} + 2{,}5\cdot 10^{-4}\tau_+^{2,5}}{1+10^{-3}\tau_+^{2,5}+\dfrac{1{,}25\cdot 10^{-3}\tau_+^3}{R_+^{1/2}}} \tag{14}$$

can be proposed as a generalizing dependence.

Results of a computation using (14) are presented in Fig. 4 by dashed lines which correspond to (9), (10), (13). It is seen that (14) agrees with the experimental data (assem-

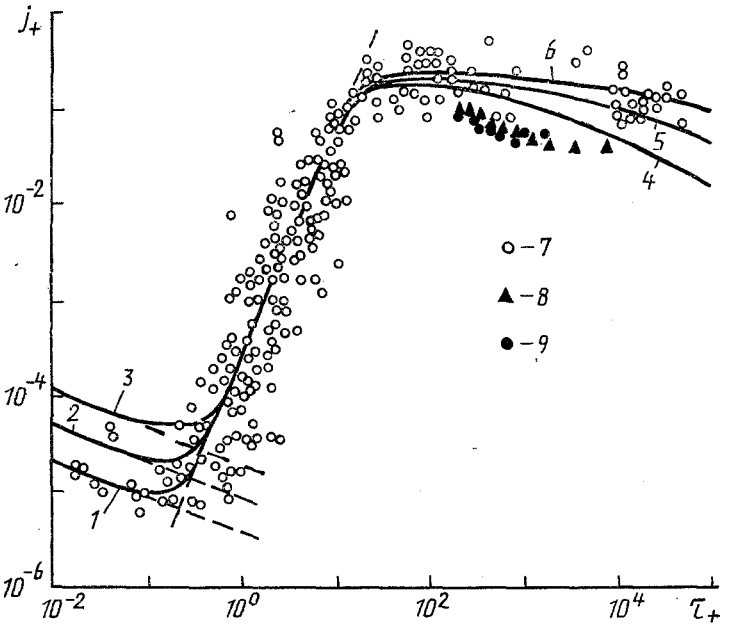

Fig. 4. Comparison of (14) with experimental data: 1) B = 10^6; 2) $3\cdot10^5$; 3) $3\cdot10^5$; 4) 10^3; 5) 10^4; 6) 10^5; 7) [14]; 8) [16]; 9) [17].

bled in [14] and supplemented by [16, 17]) in the whole range of τ_+ considered, where in conformity with the experimental data in the domain of small and large values of τ_+ it describes the stratification of the dependence $j_+(\tau_+)$ in the parameters B and R_+.

Therefore, (14) can be used to compute the deposition process in a broad range of variation of the particle inertia.

NOTATION

u_i', U_i, v_i', V_i are the fluctuating and average velocities of the gas and solid phases; $\tau = 2\rho_2 a^2/9\rho_1\nu$ is the time of particle dynamic relaxation; $D = k\Theta/6\pi\rho_1\nu a$ is the Brownian diffusion coefficient; ν is the kinematic viscosity of the gas; a is the particle radius; ρ_1, ρ_2 are the gas and solid phase densities; k is the Boltzmann constant; Θ is the temperature; r, $y = R - r$ are coordinates in the radial direction; R is the channel radius; J_w is the particle flux being deposited on the wall; u_* is the dynamic viscosity; C is the particle concentration; $C_0 = 2\int_0^1 \bar{r}C\,d\bar{r}$ is the mean particle concentration over the channel section; $\bar{y} = y/R$; $\bar{r} = r/R$; $y_+ = yu_*/\nu$. The subscript w refers to parameters at the wall.

LITERATURE CITED

1. E. P. Mednikov, Turbulent Transfer and Deposition of Aerosols [in Russian], Moscow (1981).
2. Manguturk and Sverdroop, Energetic Machines and Installations [Russian translation], 104, No. 1, 47-56 (1982).
3. A. E. Kroshilin, V. N. Kukharenko, and B. I. Nigmatulin, Izv. Akad. Nauk SSSR, Mekhan. Zhidk. Gaza, No. 4, 57-63 (1985).
4. Yao and Yang, Energetic Machines and Installations [Russian translation], 110, No. 3, 93-100 (1988).
5. I. V. Derevich and L. I. Zaichik, Izv. Akad. Nauk SSSR, Mekhan. Zhidk. Gaza, No. 5, 96-104 (1988).
6. I. V. Derevich, V. M. Eroshenko, and L. I. Zaizhik, Inzh.-Fizich. Zh., 45, No. 4, 554-560 (1983).
7. A. A. Shraiber, V. N. Milyutin, and V. P. Yatsenko, Hydrodynamics of Two-Component Flows with Solid Polydisperse Substance [in Russian], Kiev (1980).
8. B. S. Petukhov, L. G. Genin, and S. A. Kovalev, Heat Transfer in Nuclear Power Plants [in Russian], Moscow (1986).

9. P. I. Geshev, Zh. Prikl. Mekhan. Tekh. Fiz., No. 2, 61-66 (1974).
10. J. Laufer, NACA Rep. No. 1174, 1-18 (1954).
11. M. Kh. Ibragimov, V. I. Subbotin, and G. S. Taranov, Inzh.-Fizich. Zh., 19, No. 6, 1060-1069 (1970).
12. E. M. Khabakhpasheva, Thermophysics and Physical Hydrodynamics Problems [in Russian], Novosibirsk (1974), pp. 223-234.
13. S. S. Kutateladze, Near-Wall Turbulence [in Russian], Novosibirsk (1973).
14. D. D. McCoy and T. J. Hanratty, Int. J. Multiphase Flow, 3, No. 4, 319-331 (1977).
15. S. K. Friedlander and H. G. Johnstone, Indust. Eng. Chem., 49, No. 7, 1151-1156 (1957).
16. E. N. Ganic and K. Mastanaiah, Int. J. Multiphase Flow, 7, No. 4, 401-422 (1981).
17. M. M. El-Kassaby and E. N. Ganic, Int. J. Heat Mass Transfer, 29, No. 8, 1149-1158 (1986).

INVESTIGATION OF THE DEFOCUSING PROPERTIES OF A VORTICAL GAS FLOW

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Results are presented of measurements of the focal length of a gas lens formed in a swirling gas flow as well as during combustion of a glow discharge therein. An analytic dependence is obtained to estimate the focal length.

1. At this time several papers [1-3] devoted to the creation of the active medium of a CO_2 laser on the basis of a glow discharge in a swirling (vortical) gas flow have been published. Under the action of centrifugal forces and heat liberation of the discharge in such a flow, the gas density in the central domain turns out to be less than at the circumference. The optical radiation of such a flow in the gas passing along the axis of rotation acts similarly to a scattering lens. The optical strength of this lens must be known for a correct selection of the laser resonator parameters. Moreover, such an apparatus is of independent interest as a gas lens, for instance, to control the radiation of powerful lasers. It is proposed to use a swirling gas flow as a scattering lens in [4]. However, at this time there are neither theoretical nor experimental work in which the optical force of a gas lens of this kind was investigated.

Results are presented in this paper of experiments to measure the optical force of a vortical gas lens and to study its dependence on the vortex flow parameters and the power liberated in the gas by an electrical glow discharge. Theoretical relationships permitting estimation of the parameters of the gas lens that occurs are also represented.

2. Estimates of the focal length of a lenslike gas medium in a self-vacuumizing vortex tube (SVT) without a discharge can be carried out on the basis of the theory proposed in [5]. The dependence of the gas density on the radius in the vortex tube has the following form in a one-dimensional adiabatic flow approximation [5]

$$\rho = \rho_1 \left(a + b\left(\frac{r}{R}\right)^2\right)^{\frac{1}{\gamma-1}}, \tag{1}$$

where

$$a = \left(\frac{1}{\pi}\right)^{\frac{\gamma-1}{\gamma}}; \; b = \frac{\gamma-1}{2r_2^4}M^2;$$

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